Lab 2 Report

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**Introduction**

This report models and analyzes the motion of a 1 kg test mass falling in a 4 km mine shaft at the Earth’s equator. Factors such as gravitational force, drag, the Coriolis effect, and density distribution were considered to understand their impact on the falling test mass. This study has both practical and theoretical significance. In practice, understanding the behavior of objects in free fall is essential for applications in mining, geological research, and safety studies. Theoretically, it provides insights into principles of gravitation, oscillatory motion, and the relationship between orbital and fall dynamics. Python, specifically SciPy’s solve\_ivp function, was used for numerical modeling to integrate the equations of motion and simulate the falling object under various conditions.

**Calculation of fall time**

The first analysis examines how the test object falls under three scenarios. Constant gravity-no drag, variable gravity – no drag, and variable gravity with drag. The first case is the simplest model that assumes constant gravitational acceleration (g = 9.81 m/s2), the analytical solution under these conditions is t = sqrt(2h/g) = 28.57 seconds. The numerical simulation using solve\_ivp confirmed the result. According to Newton’s law of universal gravitation, gravitational acceleration varies with height and the model was updated with the gravitational relation and yielded a fall time 28.57 seconds, depicted graphically in figure 2. This is identical to our ideal no drag scenario, indicating that in the variation between a 4km tunnel and the Earth’s radius of 6,378.1km, the effects of variable gravity are minimal. The third case significantly alters fall dynamic, by using a quadratic drag model: F\_drag = -α \* v2 \*np.sign(v). α, the drag coefficient, was calibrated to produce a terminal velocity of 50 m/s. With drag included, the simulation resulted in a fall time of 35.97 seconds, approximately 26% longer than drag-free scenarios, where the test mass reached a terminal velocity before hitting the bottom of the shaft. This analysis demonstrates that while the variation in gravity over a 4km shaft has negligible impact on fall time, air resistance significantly increases the fall duration by limiting the maximum velocity.

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**Feasibility of depth measurement approach**

A logical next step was to investigate whether the Coriolis effect would affect depth measurements with a falling mass. The question is whether the Earth’s rotation could cause a lateral deviation that causes the test mass to hit the mine shaft wall before hitting the bottom. To investigate, a 2D model was implemented that incorporated vertical and lateral deviation using dvx/dt = 2ω\*vy + dragx and dvy/dt = -2ω\*vx – g + dragy, where ω is the angular velocity of the Earth. The simulations concluded that the Coriolis effect does cause lateral deviation. Without drag, the test mass deviates 4.2 meters and with drag 2.6 meters. With a shaft radius of 2.5 meters, the test mass will certainly hit the wall before reaching the bottom of the shaft in both scenarios. This indicates that the depth measurement approach is not viable due to the Coriolis effect. For deep mine shafts, particularly at the equator where the Coriolis effect is most pronounced, the depth measurement approach should either be conducted with a large shaft diameter, or a controlled guidance system. A free fall is not recommended.

**Calculation of crossing times for homogeneous and non-homogeneous Earth**

The analysis was extended to a scenario where the tunnel went through the entire Earth and was considered under a uniform density model. The gravitational force inside Earth is proportional to the distance from the center, which creates a harmonic oscillator. Figure 6 presents gravitational force profiles for different density models; Figure 7 illustrates the position and velocity of the object on Earth over time. These figures, generated by matplotlib, show that the gravitational force profile changes significantly depending on the density distribution. To get to the center of the Earth, n = 0 resulted in 1267.27 seconds while n = 9 resulted in 2831.83 seconds, that is approximately 55% faster. This makes sense because the concentrated mass at the core would create a stronger gravitational acceleration for the duration of the journey. The model was extended to a hypothetical case involving the Moon, where the results were unexpected, but knowing that the Moon’s smaller mass and radius result in a shorter crossing time despite lower average density.

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**Discussion and Future Work**

The investigation revealed key insights such as negligible gravitational acceleration in certain conditions, drag’s significant impact on fall time, in this case as much as 26%. The investigation proved that the Coriolis force makes the depth measurement approach unfeasible due to collision with the wall, and for a hypothetical Earth tunnel, the crossing time is half the orbital period for a uniform density model. The models did use several simplifications, however, such as assuming the Earth was perfectly spherical, disregarding thermal and pressure effects, and a simplified drag model. To further the work, the Earth’s ellipse shape could be considered as well as extending the analysis to other planetary bodies. This investigation provides a foundation for understanding the dynamics of objects in gravity wells, with applications ranging from mining engineering to theoretical physics and potentially even future transportation systems.